Big Ideas Math: Algebra 1

Parent Newsletter

Chapter 6: Exponential Equations and Functions

Students will...

- Simplify and evaluate square roots.
- Simplify radical expressions.
- Determine whether sums or products are rational or irrational.
- Simplify expressions using the properties of exponents.
- Simplify expressions with rational exponents.
- Identify, evaluate, and graph exponential functions.
- Solve exponential equations algebraically and graphically.
- Write, interpret, and graph exponential functions.
- Identify exponential growth and decay.
- Write, interpret, and graph exponential decay functions.
- Extend and graph geometric sequences.
- Write equations for geometric sequences.
- Solve real-life problems.
- Write the terms of recursively defined sequences.
- Write recursive equations for sequences.

Standards

Common Core: N.RN.1, N.RN.2, N.RN.3, A.REI.3, A.REI.11, F.BF.2, F.BF.3, F.IF.3, F.IF.7e, F.LE.1a, F.LE.2, A.SSE.1a, A.SSE.1b

Key Ideas

Product Property of Square Roots

- $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$, where $x, y \geq 0$.
- $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Quotient Property of Square Roots

- $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$, where $x \geq 0$ and $y > 0$.
- $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

Product of Powers Property

- To multiply powers with the same base, add their exponents.
- $4^6 \cdot 4^3 = 4^{6+3} = 4^9$
- $a^m \cdot a^n = a^{m+n}$

Power of a Power Property

- To find a power of a power, multiply the exponents.
- $\left(4^6\right)^3 = 4^{6 \cdot 3} = 4^{18}$
- $\left(a^m\right)^n = a^{mn}$

Power of a Quotient Property

- To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.
- $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$.

Quotient of Powers Property

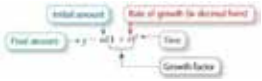
- To divide powers with the same base, subtract their exponents.
- $\frac{4^6}{4^3} = 4^{6-3} = 4^3$
- $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

Power of a Product Property

- To find a power of a product, find the power of each factor and multiply.
- $(3 \cdot 2)^5 = 3^5 \cdot 2^5$
- $(ab)^m = a^m b^m$

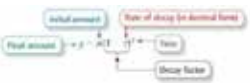
Exponential Growth Functions

A function of the form $y = (1 + r)^t$, where $a > 0$ and $r > 0$, is an exponential growth function.



Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an exponential decay function.



Compound Interest

The interest earned on the principal *and* on previously earned interest. The balance y on an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

Geometric Sequence

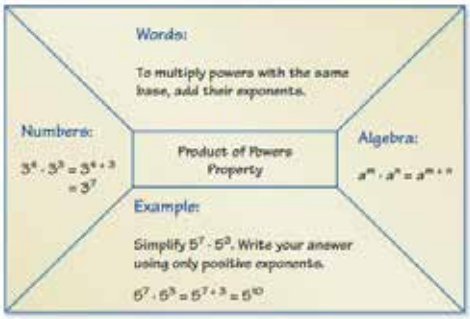
In a geometric sequence, the ratio between consecutive terms is the same. This ratio is called the common ratio. Each term is found by multiplying the previous term by the common ratio.

Equation for a Geometric Sequence

Let a_n be the nth term of a geometric sequence with first term a_1 and common ratio r. The nth term is given by $a_n = a_1 r^{n-1}$.

Reference Tools

An **Information Frame** can be used to help students organize and remember concepts. Students write the topic in the middle rectangle. Then students write related concepts in the spaces around the rectangle. Students can place their information frames on note cards to use as a quick study reference.



Essential Questions

How can you multiply and divide square roots?

How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

How can you write and evaluate an *n*th root of a number?

What are the characteristics of an exponential function?

What are the characteristics of exponential growth?

What are the characteristics of exponential decay?

How are geometric sequences used to describe patterns?

Key Terms

A set of numbers is ***closed*** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set.

When $b^n = a$ for an integer n greater than 1, b is an ***nth root*** of a.

A function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$ is an ***exponential function***.

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an ***exponential growth function***.

Compound interest is interest earned on the principal and on the previously earned interest.

Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an ***exponential decay function***.

In a ***geometric sequence***, the ratio between consecutive terms is the same. This ratio is called the ***common ratio***.

Quick Review

- To simplify a square root expression, perform any operations and factor to remove any perfect square factors other than 1 from the radicand.
- Because $\sqrt{n} = n^{1/2}$, you can generalize that $\sqrt[n]{a} = a^{1/n}$.
- An exponential equation that cannot easily be rewritten in the form $b^x = b^y$ can be solved by graphing.
- You can use a table to organize the terms in a geometric sequence.
- You can find the common ratio by finding the ratio of a term to the previous term.

What's the Point?

The ability to understand and use recursive sequences is very useful in real life. For example, programmers write the codes that are needed for applications you use each day. Some code contains recursive relationships. Recursion also occurs in nature. Biologists use many different recursion models for plant growth.

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 6: Mathematical Recursion STEM Video is available online at www.bigideasmath.com.